FREEZING OF WATER IN VERTICAL CHANNELS FORMED IN A FROZEN SOIL LAYER

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A determination is made of the maximum possible pressure in a water bubble located in the nonuniform temperature field of surrounding frozen soil which becomes permeable as it melts. The principles found are used to explain a number of natural processes.

During industrial exploitation of petroleum and natural gas sites, long vertical channels are placed in the earth, for example, the annular spaces between a casing and the walls of the well. Such spaces cannot always be filled with solid matter for technical or economic reasons, and they become filled with water. In Soviet near eastern regions this water freezes, primarily in segments which border the coldest layers of the frozen depths, and in winter, also at the boundary with the atmosphere. As a result a long closed volume of water is formed in the channel, which can generate a pressure high enough to damage adjacent construction, for example, the well pipe.

The nonuniform temperature distribution of the medium surrounding the water bubble does not always allow use of known methods to determine the pressure within the bubble [1, 2], since under certain conditions walls formed by regions of sandy soil allow loss of hermetic sealing. In fact, because of rapid temperature equalization within the bubble, the water in it freezes only in those segments opposite which the temperatures of the surrounding medium are the lowest, while at points where the temperature of the surrounding medium is higher, pore ice melts under the water pressure within the bubble. A special case was considered in [3] with the assumption that the bubble occupies a planar slot, while in practice the case of annular slots bounded on the inside by the well pipe and on the outside by the frozen soil is no less important. Water pressure in such slots is created by winter growth of the ice mass coating the upper orifice. The lower end of the bubble rests on a cement ring or an ice mass formed in the warm period of the year. Pressure release in the bubble occurs because of melting of pore ice in sandy (or suspensionlike) strata, as well as thawing of the lower ice mass if such exists. These two cases differ from each other in the geometry of thermal flux propagation (plane-radial and plane-parallel, respectively), and will be considered separately. The conclusions obtained by solving the second problem will be used to explain experimentally observed phenomena, the mechanisms of which are yet to be well defined. Among these are migration of clay particles in ice in the direction of increasing temperature [4] and water flow through ice plates [5].

1. For quantitative study of the effect of pressure release in an annular water bubble by thawing of pore ice in sandy strata (Fig. 1) we assume the net capacity of the strata is h, the mean porosity of the thawed portion is m, and that the annular gap rests below on a cement cup. It is limited on the inside by a column of radius r_c, and on the outside by the wall with radius r_w. Thus its cross sectional area is equal to $A = \pi(r_c^2 - r_w^2)$. Pressure is created within the bubble by growth in the ice mass at the surface of the earth, the thickness of which at time t is equal to $\ell(t)$. The temperature on the surface is equal to T_0, and the initial temperature of the permeable sandy strata is T_i. The current temperature value in the water bubble is T_m and the corresponding equilibrium pressure depends on time, while $T_0 < T_m < T_i$.

For simplicity we will assume that the water in the annular gap and the thawed portion of the stratum $r_w \leq r \leq r_m(t)$ have identical temperatures. This simplification leads to some reduction in pressure in the gap due to neglect of filtration resistance. However if in this formulation it can be shown that warping of the column is possible, the situation

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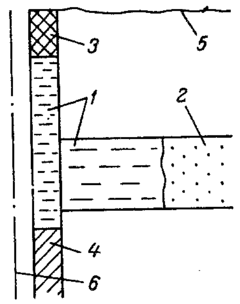

Fig. 1. Diagram of annular slit surrounding well in frozen soil: 1) water in annular gap and thawed portion of stratum; 2) frozen portion of porous stratum; 3) ice mass; 4) cement column or lower ice mass; 5) surface of earth; 6) well axis.

will be more severe when this assumption is eliminated.

The rate of motion of the ice—water boundary $r_m(t)$ in the stratum is determined simultaneously by the quantity of water entering it from the annular reservoir and the intensity of heat flux to the front from the exterior ($r_m \leq r < \infty$) region of the stratum.

The first of these conditions is expressed by the equation

$$A\frac{dl}{dt}=2\pi hm\left(r\frac{dr}{dt}\right)_{r_m}, \tag{1}$$

while the second is given by

$$\lambda\left(r\frac{\partial T}{\partial r}\right)_{r_m}=\rho_i Lm\left(r\frac{dr}{dt}\right)_{r_m}. \tag{2}$$

With no great error it can be assumed that the temperature distribution in the ice mass is quasisteady state, whereupon the current thickness of the mass $\ell(t)$ and the water temperature in the annular gap T_m are related by the thermal balance condition

$$\lambda_i\frac{T_m-T_0}{l}=\rho_i L\frac{dl}{dt}. \tag{3}$$

These three conditions permit definition of the three unknown quantities appearing therein: r_m, ℓ, and T_m, if we can find the temperature distribution in the frozen portion of the stratum $r_m \leq r \leq \infty$, the thermal conductivity and diffusivity of which are equal to λ and α, respectively. The temperature field in the stratum can be determined using a fictitious direct acting heat source with unknown intensity $q(t)$ located at the origin of the coordinate system $r = 0$:

$$T(r,t)=T_i-\int_0^t q_1(t-u)\exp\left(-\frac{r^2}{4\alpha u}\right)\frac{du}{u}. \tag{4}$$

To eliminate the additional unknown function $q_1(t)$ we have the condition

$$T_i-T_m=\int_0^t q_1(t-u)\exp\left(-\frac{r_m^2}{4\alpha u}\right)\frac{du}{u}. \tag{5}$$

Introducing the dimensionless variables

$$\tau = \frac{\alpha_i t}{r_w^2};\quad \eta = \frac{r_m}{r_w};\quad x = \frac{l}{r_w};\quad q(\tau) = \frac{\lambda_i}{\alpha_i \rho_i L} q_1(t)$$

and parameters

$$K = \frac{\lambda_i (T_i - T_0)}{\alpha_i \rho_i L};\quad M = \frac{r_w^2 - r_c^2}{2hmr_w};\quad N = \frac{m\lambda_i}{\lambda};\quad \beta = \frac{\alpha_i}{\alpha},$$

after eliminating T_m from Eqs. (2)-(5), for determination of the three remaining unknowns x, η and q(τ) we obtain the following system of three equations:

$$K = x\frac{dx}{d\tau} + \int_0^\tau q(\tau - \sigma)\exp\left(-\frac{\beta\eta^2}{4\sigma}\right)\frac{d\sigma}{\sigma}, \tag{6}$$

$$MN\frac{dx}{d\tau} = \frac{1}{2}\beta\eta^2\int_0^\tau q(\tau - \sigma)\exp\left(-\frac{\beta\eta^2}{4\sigma}\right)\frac{d\sigma}{\sigma^2}, \tag{7}$$

$$\eta^2 = 1 + 2Mx. \tag{8}$$

This system of integrodifferential equations is analogous to that found in calculation of underground gas reservoirs and horizontal aquifers, and can be solved by the method developed by Charnyi [6], consisting of the following. Let τ take on discrete values on a uniform grid, with x_k, η_k being the values of the corresponding functions at the point τ_k, while q_k is the mean value of the function q(τ) in the interval (τ_{k-1}, τ_k). Then, using the theorem of the mean to calculate the integrals in Eqs. (6), (7) in each of the intervals and replacing the derivatives on the left sides of the expressions by finite differences, we obtain:

$$K = \frac{1}{2}\frac{x_n^2 - x_{n-1}^2}{\tau_n - \tau_{n-1}} = \sum_{k=1}^{n} q_{n-k+1}\left[E_1\left(\frac{\beta\eta_{n-1}^2}{4\tau_k}\right) - E_1\left(\frac{\beta\eta_{n-1}^2}{4\tau_{k-1}}\right)\right];$$

$$\frac{1}{2}MN\frac{x_n - x_{n-1}}{\tau_n - \tau_{n-1}} = \sum_{n=1}^{n} q_{n-k+1}\left[E_0\left(\frac{\beta\eta_{n-1}^2}{4\tau_k}\right) - E_0\left(\frac{\beta\eta_{n-1}^2}{4\tau_{k-1}}\right)\right];$$

$$\eta_n^2 = 1 + 2Mx_n,$$

where for brevity we have introduced the notation

$$E_n(x) = \int_x^\infty e^{-x}\frac{dx}{x^n}.$$

For n = 1 only one term containing q_1 as a factor remains. Since $x_0 = 1$, $\eta_0 = 1$ the system rapidly gives x_1 and q_1. We will now assume that x_k, η_k, q_k are defined for all $k \le n - 1$. Then the unknowns x_n, q_n are again determined from the system by the same method.

Results of solving system (6)-(8) are presented in Fig. 2. Three different values were chosen for the parameter M, which defines the ratio of the ice mass face area to the area of the sandy stratum filtration surface on the well wall: M = 1, 0.1, 0.01. The parameters K, N, β were not varied and their values corresponded to mean northern Tyumen' region conditions: K = 0.1, N = 0.3, β = 0.7. As is evident from Fig. 2, with decrease in M there is an insignificant increase in the rate of growth of the ice mass and simultaneous abrupt increase in the parameter $\theta = (T_m - T_0)/(T_i - T_0)$. Thus, the larger the sandy stratum input filtration surface, the closer the final temperature in the gap will be to its initial value. The maximum calculated pressures are observed at the very start of the process, although from a practical viewpoint they are not dangerous, since they usually lead to breakage of the ice mass rather than distortion of the wall pipe.

Thus, if pressure growth occurs in a vertical channel its maximum value should be determined from the temperature of the permeable stratum of highest power. This conclusion

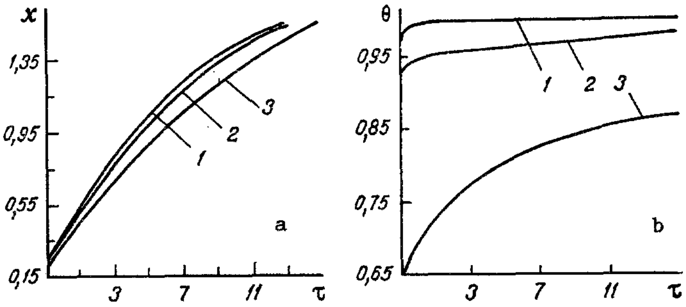

Fig. 2. Dynamics of increase in thickness of ice mass x (a) and relative dimensionless temperature θ in annular gap (b) for various ratios of gap area to input surface of porous stratum, defined by parameter M: 1) M = 0.01; 2) 0.1; 3) 1.

corresponds completely with that made previously in [3] with simplifying assumptions.

The temperature of the frozen depths in the north of western Siberia gradually decreases with movement upward from below and the minimum constant value is reached near the neutral layer. Above this layer it oscillates about this constant level depending on the season. If the equilibrium pressure for the temperature of the neutral layer is higher in value than that required to distort the pipe, and moreover the throat of the well is covered by an ice mass before the onset of winter cold, then the pipe can be damaged even in the presence of sandy layers. On the other hand, if this seal does not exist, then the pressure can be released into some porous stratum near the base of the frozen layer where the temperature is close to 0°C.

2. We will assume that the annular gap is supported at its bottom not by a cement cup, but by an ice mass. Upon increase in pressure in the gap the mass will melt from its upper face, introducing another contribution to pressure release. In order to estimate the value of this contribution, we will assume that the lateral walls of the gap are coated by adiabatic and impermeable barrier, as a result of which the problem becomes planar.

To solve this problem we direct the x-axis vertically downward, taking as the origin the point on the earth's surface where at time t = 0 a constant negative temperature T_0 is established, much lower than the phase transition temperature for water. Moreover, let T_i be the initial temperature of the lower ice mass. If the latter has sufficient thickness it can be considered infinite with no great error. In the process of freezing the lower face of the upper ice mass $x_m(t)$ will increase, the pressure P_m developed in a water layer of thickness δ is transferred to the upper face of the lower mass $x_s(t)$, decreasing the melting point of the adjacent ice layer. Thus, in both the water and the faces of the two ice masses facing each other a temperature T_m is established, which is in equilibrium with the pressure in the water layer P_m. In view of the low compressibility of water the thickness of this layer practically always remains constant.

The temperature field in the upper and lower masses $T_1(x, t)$ and $T_2(x, t)$ can be represented in the form:

$$\frac{T_m - T_1}{T_m - T_0} = 1 - \operatorname{erf} \frac{x}{2\sqrt{\alpha_i t}} \Big/ \operatorname{erf} \frac{x_m}{2\sqrt{\alpha_i t}}; \tag{9}$$

$$\frac{T_2 - T_m}{T_m - T_i} = -1 + \operatorname{cerf} \frac{x - \delta}{2\sqrt{\alpha_i t}} \Big/ \operatorname{cerf} \frac{x_s - \delta}{2\sqrt{\alpha_i t}}, \tag{10}$$

while thermal balance on the moving faces of the ice masses is represented by the equations:

$$x = x_m(t);\ \lambda_i = \frac{\partial T_1}{\partial x} = \rho_i L \frac{dx_m}{\partial t};\quad x = x_s(t);\quad \lambda_i \frac{\partial T_2}{\partial x} = \rho_i L \frac{dx_s}{dt}.$$

Taking $x_m = 2a\sqrt{\alpha_i t}$, $x_s = \delta + 2a\sqrt{\alpha_i t}$, where a is a constant but unknown quantity, we obtain from Eqs. (9), (10) the following equations:

$$\sqrt{\pi}\, a \exp(a^2) \operatorname{erf} a = K \frac{T_m - T_0}{T_i - T_0}; \quad (11)$$

$$\sqrt{\pi}\, a \exp(a^2) \operatorname{cerf} a = K \frac{T_i - T_m}{T_i - T_0}. \quad (12)$$

These serve to determine the temperature in the water layer T_m and the parameter a, and together with the latter, the law of downward motion of the water layer. To find the latter by addition of Eqs. (11) and (12) we obtain the simple expression

$$\sqrt{\pi}\, a \exp(a^2) = K. \quad (13)$$

In the processes under study K $\approx$ 0.1, and for such small values with only slight error from Eq. (13), and then from Eq. (11), we have

$$a \simeq K/\sqrt{\pi}; \quad \frac{T_m - T_0}{T_i - T_0} \simeq \frac{2}{\pi} K. \quad (14)$$

From this last expression it is evident that in the range of the parameter K studied the temperature in the water bubble differs little from the air temperature, and thus, fusion of the lower ice mass at its upper face has practically no effect on the water pressure in the annular gap.

This problem is of interest because it permits a new approach to some physical phenomena.

We will use Eq. (14) to evaluate the speed of the water bubble motion. For ice we take $\rho_i = 0.917 \cdot 10^3$ kg/m³, $L = 334.5 \cdot 10^3$ J/kg, $\lambda_i = 2.213$ W/(m·K), $\alpha = 1.14 \cdot 10^{-6}$ m²/sec. Then for $\Delta T = T_i - T_0 = 10$°C we obtain $K = 6.329 \cdot 10^{-2}$, $a = 3.575$, so that over a day the bubble moves 2.24 cm, over a month, 67.3 cm. It is important to note that this velocity does not depend on the length of the water bubble in the gap. Thus if in some manner or other we introduce into the thickness of an ice mass across the ends of which a temperature head $\Delta T = 10$°C has been created a droplet of water, it will move in the direction of the higher temperature with approximately the same velocity as found above, with a necessary correction for the spatial character of heat transfer. However the qualitative picture is also produced by our simplified formulation.

A water droplet can be naturally frozen into ice if the drop contains a clay particle. For a long period it remains immobile until the adsorption pressure in the layer of weakly bound water equals the pressure in equilibrium with the temperature at the point where the particle is located. As soon as the outer layers of the water begin to freeze the droplet displaces, following the isotherm corresponding of adsorption pressure. Thus the ice is spontaneously purified of initially frozen clay particles, as was observed in the experiments of [4], but not explained.

The results obtained herein permit explanation of another phenomenon which Miller [5] termed the sandwich-effect.

We will first assume that within the framework of the problem considered above the pressure on the water bubble is created by a piston. Then the ice begins to melt as soon as the ice pressure exceeds the equilibrium value for its initial temperature. The heat required for ice fusion is obtained from the depths of the ice mass. Let the latter have a finite thickness and be supported below by a layer of water in which no pressure is created. Then the heat from fusion of the upper face of the ice mass will be removed below the lower face. As a result the ice mass will begin to grow on the bottom, driving back the water layer located below it. Thus a water flow is created in the system although it is divided into two parts by an impermeable ice mass.

Miller observed this phenomenon in a chamber which was divided into two halves by an ice plate held between two metallic grids. The water in the chamber was cooled to −(0.07-0.15)°C, and when pressure was created in one half, the level increased in the other. Thus, at a pressure of 1.306 bar (980 mm Hg) the water flow rate through the ice seal comprised 0.555 cm/sec according to Miller's data. It is obvious that this velocity is only an apparent one and the water flow will only continue while the ice seal covers the space between the grids.

NOTATION

r, radial distance from well axis; r_c, r_w, radii of column and well; r_m, radius at which melting phase transition occurs; h, m, thickness and porosity of stratum; ℓ, thickness of ice mass; t, time; λ_i and α_i, λ and α, thermal conductivities and diffusivities of ice and porous stratum; ρ_i, L, density and heat of fusion of ice; T_0, T_i, T_m, temperature of earth's surface, porous stratum, and water bubble; $q_1(t)$ and $q(\tau)$, intensity of fictitious heat source, dimensioned and dimensionless; τ, η, x, dimensionless time, thawing radius, and thickness of ice mass; K, M, N, β, α, dimensionless parameters; u, σ, integration variables; δ, length of water bubble; $x_m(t)$, $x_s(t)$, coordinates limiting bubble.

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FINITE-DIFFERENCE SOLUTION OF THE OPTIMIZATION PROBLEM IN HIGH-SPEED HEATING OF A BODY OF SIMPLE SHAPE BY INTERNAL HEAT SOURCES

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A method is proposed for construction of optimal fast-response control of body heating under constraints on the control (internal heat sources) and the temperature field or stress-strain parameters.

Body heating by internal heat sources occurs in modern technological processes, for instance, the induction heating of articles by high-frequency currents [1], in heat exchanger elements [2], in chemical and nuclear reactions [3], etc. Among analogous processes can also be the heating of thin-walled elements during convective heat transfer since in this case the temperature of the external medium is in the right side of the heat-conduction equations [4].

The optimization of body heating relative to fast-response is of direct practical interest to raise the productivity of heater plants [5]. In connection with the limited power of the installation, here, as well as taking into account the requirement of material strength and possibilities of intensive fusion, oxidation, phase microstructure transformation and other phenomena that take place at high temperatures in metals and many materials, constraints are imposed on the control actions, the thermal process parameters, and the stress-strain state [6].

Let us consider the problem of constructing an optimal fast-response control of the heating of homogeneous or inhomogeneous plates, hollow cylinders and spheres by internal heat sources under constraints on the control, the body temperature, the temperature drop, and the thermoelastic stresses in the body.

Heating in the above-mentioned bodies is described by the following boundary-value problems:

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